Bohr compactification and Chu duality of non-abelian locally compact groups

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Salvador Hernández

Universitat Jaume I

It is a consequence of the celebrated Gel'fand and Raĭkov Theorem that the set of all unitary representations of a locally compact group G contains the information necessary to recover the topological and algebraic structure of the group. However, in general, such representations may not be finite dimensional. The result is that if we form a *dual space* associated to the set of all representations, we obtain an object whose structure is very involved. Chu, motivated by the work of Hochschild and Mostow for compact groups, considered the groups with enough finite dimensional representations to separate the points (the so-called maximally almost periodic groups) and established a duality theory within this class that extends both Pontryagin and Tannaka-Kreĭn dualities for locally compact Abelian groups and compact groups respectively.

A pair (bG, b), where bG is a compact group and $b\colon G \to bG$ is a continuous group homomorphism , is said to be the *Bohr compactification* of G if the following universal property holds: for every continuous homomorphism h from G into a compact group K there is a continuous homomorphism h^b from bG into K that extends h in the sense that $h = h^b \circ b$. The Bohr compactification (bG, b) is the unique (up to equivalence) largest compactification of G. Although, for locally compact abelian groups the Bohr compactification is a big monster, for non-Abelian groups the situation is much more interesting and one can say that all options are possible. Here we discuss the relation between the notions of duality and the Bohr compactification of non-Abelian locally compact groups. Our main result establishes the existence of noncompact Chu reflexive groups whose partial dual spaces $\widehat{G}_n$, equipped with the Fell topology, are discrete. Furthermore, we characterize when the Bohr compactification of a locally compact group is topologically isomorphic to its Chu or unitary quasi-dual.

Joint work with María V. Ferrer