On the Grothendieck property for Banach spaces $Lip_0(M)$ of Lipschitz functions

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Jerzy Kąkol

Department of Mathematics and Informatics, Adam Mickiewicz University

We study the following general question: For which metric spaces M, in particular for which Banach spaces M, do the spaces $lip_0(M)$ have the Grothendieck property? Recall that a Banach space E is called *Grothendieck* if every weak* convergent sequence in the dual space E^* converges weakly. Typical examples of Grothendieck spaces are reflexive spaces, the space ℓ_∞ or more generally spaces $C(K)$ over extremally disconnected compact spaces K, the space H^∞ of all bounded analytic functions on the unit disk and von Neumann algebras. It seems that apart from $Lip_0([0,1]) \simeq \ell_\infty \simeq Lip_0(2^{\mathbb{N}})$ there is no known example of a Banach space $Lip_0(M)$ which is a Grothendieck space. We collect a number of conditions for metric spaces M implying that the corresponding spaces $Lip_0(M)$ are not Grothendieck. For example, if a Banach space E is a $C(K)$-space, $L_1(\mu)$-space, $lip_0(M)$-space, or $\mathcal{F}(M)$-space, then $Lip_0(E)$ is not Grothendieck. Open questions are provided.