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w^*-w points of continuity of the dual unit ball of a Banach space

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If X is a Banach space, and $B(X^*)$ ($S(X^*)$) denotes its dual unit ball (respectively, its dual unit sphere), then a point $x^* \in S(X^*)$ is of continuity of $\mathrm{id} : (B(X^*), w^*) \to (B(X^*), w)$ (we call it a **point of w-w^*-continuity**) if and only if x^* has a unique Hahn–Banach extension to X^{**} [G. Godefroy]. The space X is said to be **M-embedded** if it is an M-ideal in X^{**}. If this is the case, then every point of $S(X^*)$ is of w^*-w-continuity; if, moreover, X is nonreflexive, then no point of $B(X^{**})$ is of w^*-w-continuity. If X is a Banach space such that X^{**}/X is separable and non-reflexive, then there is a renorming Z of X and a point of w^*-w-continuity in $B(Z^*)$ that is not of w^*-w-continuity in $B(Z^{***})$. We analyze the structure of the set of all points of w^*-w-continuity for non-reflexive Hahn–Banach smooth spaces. We apply these and other results to the space $\mathcal{K}(\ell^p)$, to von Neumann algebras, and to $L^1(\mu)$ spaces. This is a joint work with S. Daptari and T.S.S.R.K. Rao.