Compactness of the Weyl operator in $\mathcal{S}_\omega$

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In this talk we study the classical Weyl operator in the frame of spaces of ultra-differentiable rapidly decreasing functions $\mathcal{S}_\omega(\mathbb{R}^d)$, where ω is a (non-quasianalytic) weight function in the sense of Braun-Meise-Taylor. For $a \in \mathcal{S}'_\omega(\mathbb{R}^{2d})$, the Weyl operator $a^w(x, D)$ with symbol a applied to $f \in \mathcal{S}_\omega(\mathbb{R}^d)$ is the distribution defined by

$$\langle a^w(x, D)f, g\rangle = (2\pi)^{-d}\langle a, \mathrm{Wig}\,(g, f)\rangle, \quad g \in \mathcal{S}_\omega(\mathbb{R}^d),$$

where $\mathrm{Wig}\,(g, f) = \int_{\mathbb{R}^d} e^{-iy\xi} g(x + y/2)\overline{f(x - y/2)}\, dy$ is the Wigner transform of g and f. We characterize, using techniques from time-frequency analysis, continuity and compactness of the Weyl operator

$$a^w(x, D) : \mathcal{S}_\omega(\mathbb{R}^d) \to \mathcal{S}_\omega(\mathbb{R}^d)$$

in terms of the short-time Fourier transform of the symbol a. As a consequence, we give results about the compactness of the localization operator in $\mathcal{S}_\omega$, in relation with the spaces of ω-multipliers and ω-convolutors of $\mathcal{S}_\omega$.

The talk is based on a collaboration with V. Asensio, C. Boiti, and D. Jornet, cf. [1].

References

[1] V. Asensio, C. Boiti, D. Jornet, A. Oliaro, On the compactness of the Weyl operator in $\mathcal{S}_\omega$, *J. Math. Anal. Appl.*, **546** (2025), 129214.