Smooth extension from closed sets via curve-based criteria

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A fundamental theorem of Boman asserts that a real-valued function defined on an open subset of $\mathbb{R}^n$ is of class $\mathcal{C}^\infty$ if and only if its composition with every $\mathcal{C}^\infty$ curve is $\mathcal{C}^\infty$. In this talk, I will explore the extent to which this characterization remains true for functions defined on closed subsets of $\mathbb{R}^n$. In particular, I will show that if a function defined on a closed set with Hölder boundary or a closed fat subanalytic set (under a natural topological condition) is smooth along all smooth curves, then it admits a $\mathcal{C}^\infty$ extension to the ambient space $\mathbb{R}^n$. There is a precise quantitative relationship between the sharpness of boundary singularities of the domain and the loss of regularity in the determination of the derivatives at boundary points. Similar results also hold in the real analytic and ultradifferentiable setting. Time permitting, I will discuss functional-analytic properties of the associated function spaces, highlighting, in particular, that they satisfy appropriate exponential laws.

References

[1] A. Rainer, Arc-smooth functions on closed sets, *Compos. Math.*, **155** (2019), 645–680.

[2] A. Rainer, Arc-smooth functions and cuspidality of sets, *J. Anal. Math.* (2024), https://doi.org/10.1007/s11854-024-0337-0.

[3] A. Rainer, On real analytic functions on closed subanalytic domains, *Arch. Math. (Basel)* **122** (2024), 639–650.

[4] A. Rainer, On spaces of arc-smooth maps, arXiv:2503.07023.