Surjectivity of the asymptotic Peano-Borel map

(in Carleman ultraholomorphic classes defined by

weight sequences with shifted moments)

Javier Jiménez-Garrido (Univ. Cantabria)
joint work with
I. Miguel-Cantero (Univ. Valladolid),
J. Sanz (Univ. Valladolid),
G. Schindl (Univ. Vienna).

June 2025 - WFAJB70, Valencia, Spain

WFAV 2015

WFAV 2015

WFAV 2015

- J. Jiménez-Garrido, I. Miguel-Cantero, J. Sanz, G. Schindl, Surjectivity of the asymptotic Borel map in Carleman ultraholomorphic classes defined by sequences with shifted moments, arXiv:2505.09444
- J. Jiménez-Garrido, I. Miguel-Cantero, J. Sanz, G. Schindl, Stability properties of ultraholomorphic classes of Roumieu-type defined by weight matrices, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **118** (2024).
- J. Jiménez-Garrido, D. N. Nenning and G. Schindl, On generalized definitions of ultradifferentiable classes, J. Math. Anal. Appl. **526** (2023).
- J. Jiménez-Garrido, I. Miguel-Cantero, J. Sanz, G. Schindl, Optimal flat functions in Carleman-Roumieu ultraholomorphic classes in sectors, Results Math. (2023).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Equality of ultradifferentiable classes by means of indices of mixed O-regular variation, Results Math. **77** (2022).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Surjectivity of the asymptotic Borel map in Carleman-Roumieu ultraholomorphic classes defined by regular sequences, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **115** (2021).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Sectorial extensions for ultraholomorphic classes defined by weight functions, Math. Nachr. **293** (2020)
- J. Jiménez-Garrido, J. Sanz and G. Schindl, A Phragmén-Lindelöf theorem via proximate orders, and the propagation of asymptotics, J. Geom. Anal. **30** (2020).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Ultraholomorphic extension theorems in the mixed setting, Banach J. Math. Anal. **14** (2020).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, The surjectivity of the Borel mapping in the mixed setting for ultradifferentiable ramification spaces, Monatsh. Math. **191** (2020).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Indices of O-regular variation for weight functions and weight sequences, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **113** (2019).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Sectorial extensions, via Laplace transforms, in ultraholomorphic classes defined by weight functions, Results Math. **74** (2019).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Injectivity and surjectivity of the asymptotic Borel map in Carleman ultraholomorphic classes, J. Math. Anal. Appl. **469** (2019).
- J. Jiménez-Garrido, J. Sanz and G. Schindl, Log-convex sequences and nonzero proximate orders, J. Math. Anal. Appl. **448** (2017).

Surjectivity of the asymptotic Peano-Borel map

(in Carleman ultraholomorphic classes defined by weight sequences with shifted moments)

Javier Jiménez-Garrido (Univ. Cantabria)
joint work with
I. Miguel-Cantero (Univ. Valladolid),
J. Sanz (Univ. Valladolid),
G. Schindl (Univ. Vienna).

June 2025 - WFAJB70, Valencia, Spain

Weight sequences and sectors

$\boldsymbol{M} = (M_n)_{n=0}^{\infty}$ is a sequence of positive real numbers with $M_0 = 1$.

$\boldsymbol{M}$ is a **weight sequence (w.s.)**: if $\boldsymbol{M}$ is **logarithmically convex (lc)**, i.e., for all $n \in \mathbb{N}_{\geq 1}$: $M_n^2 \leq M_{n-1}M_{n+1}$, and if $\lim_{n\to\infty} M_n^{1/n} = \infty$.

The **sequence of quotients** is $\boldsymbol{m} = (m_n := M_{n+1}/M_n)_{n=0}^{\infty}$.

$\boldsymbol{M}$ is w.s. if and only if $\boldsymbol{m}$ is nondecreasing and $\lim\limits_{n\to\infty} m_n = \infty$.

$\mathcal{R}$ stands for the Riemann surface of the logarithm. For $\gamma > 0$, we write $S_\gamma := \{z \in \mathcal{R} : |\text{arg}(z)| < \frac{\gamma\,\pi}{2}\}$ and we write S for any sector.

Uniform $\boldsymbol{M}$-asymptotic expansion

Given $h>0$, we say that $f\in\mathcal{H}(S)$ admits $\widehat{f}=\sum_{k\geq 0}a_kz^k\in\mathbb{C}[[z]]$ as its **uniform $\boldsymbol{M}$-asymptotic expansion in S (of type $1/h$)** if

$$\exists C>0:\ \forall n\geq 0,\ \forall z\in S, \left|f(z)-\sum_{k=0}^{n-1}a_kz^k\right|\leq Ch^nM_n|z|^n.$$

The space of functions admitting uniform $\boldsymbol{M}$-asymptotic expansion of type $1/h$ in S is denoted by $\widetilde{\mathcal{A}}^u_{\boldsymbol{M},h}(S)$ and it is endowed with the norm

$$\|f\|_{\boldsymbol{M},h,\widetilde{u}}:=\sup_{z\in S,n\geq 0}\frac{|f(z)-\sum_{k=0}^{n-1}a_kz^k|}{h^nM_n|z|^n},$$

which makes it a Banach space.

Carleman classes

Given $h>0$, we say that $f\in\mathcal{H}(S)$ belongs to the class $\mathcal{A}_{\boldsymbol{M},h}(S)$ if

$$\|f\|_{\boldsymbol{M},h}:=\sup_{z\in S,n\geq 0}\frac{|f^{(n)}(z)|}{h^n M_n}<\infty,$$

and $(\mathcal{A}_{\boldsymbol{M},h}(S),\|\cdot\|_{\boldsymbol{M},h})$ is a Banach space

Given $h>0$, the class of **formal power series** is defined by

$$\mathbb{C}[[z]]_{\boldsymbol{M},h}=\Big\{\widehat{f}=\sum_{n=0}^{\infty}a_nz^n\in\mathbb{C}[[z]]:\ \Big|\widehat{f}\Big|_{\boldsymbol{M},h}:=\sup_{n\geq 0}\frac{|a_n|}{h^nM_n}<\infty\Big\},$$

and $(\mathbb{C}[[z]]_{\boldsymbol{M},h},|\cdot|_{\boldsymbol{M},h})$ is a Banach space

Spaces of Roumieu and Beurling type

We will consider the **Roumieu-type** spaces,

$$\widetilde{\mathcal{A}}^u_{\{\boldsymbol{M}\}}(S) := \bigcup_{h>0} \widetilde{\mathcal{A}}^u_{\boldsymbol{M},h}(S),$$

and, similarly, $\mathcal{A}_{\{\boldsymbol{M}\}}(S)$ and $\mathbb{C}[[z]]_{\{\boldsymbol{M}\}}$, which are (LB) spaces.

We will also consider the **Beurling-type** spaces

$$\widetilde{\mathcal{A}}^u_{(\boldsymbol{M})}(S) := \bigcap_{h>0} \widetilde{\mathcal{A}}^u_{\boldsymbol{M},h}(S),$$

and, similarly, $\mathcal{A}_{(\boldsymbol{M})}(S)$ and $\mathbb{C}[[z]]_{(\boldsymbol{M})}$ which are Fréchet spaces.

We write $\mathcal{A}_{[\boldsymbol{M}]}(S)$, $\widetilde{\mathcal{A}}^u_{[\boldsymbol{M}]}(S)$ or $\mathbb{C}[[z]]_{[\boldsymbol{M}]}$, when a statement is valid for both Roumieu type and Beurling type.

Asymptotic Peano-Borel map

Let $\boldsymbol{M}$ be a sequence, $\widehat{\boldsymbol{M}} = (n!M_n)_{n=0}^{\infty}$, $h > 0$, and S be a sector.

(a) The inclusion map $\mathcal{A}_{\widehat{\boldsymbol{M}},h}(S) \hookrightarrow \widetilde{\mathcal{A}}^u_{\boldsymbol{M},h}(S)$ is continuous. Consequently, $\mathcal{A}_{[\widehat{\boldsymbol{M}}]}(S) \hookrightarrow \widetilde{\mathcal{A}}^u_{[\boldsymbol{M}]}(S)$ is continuous.

(b) If T is a proper subsector of S, then there exists $c = c_{T,S} > 0$, such that the restriction map from $\widetilde{\mathcal{A}}^u_{\boldsymbol{M},h}(S)$ to $\mathcal{A}_{\widehat{\boldsymbol{M}},ch}(T)$ is continuous and it is also continuous from $\widetilde{\mathcal{A}}^u_{[\boldsymbol{M}]}(S)$ to $\mathcal{A}_{[\widehat{\boldsymbol{M}}]}(T)$.

The **asymptotic Peano-Borel map** $\widetilde{\mathcal{B}}$ sends a function $f \in \widetilde{\mathcal{A}}^u_{\boldsymbol{M},h}(S)$ into its $\boldsymbol{M}$-asymptotic expansion $\widehat{f} \in \mathbb{C}[[z]]_{\boldsymbol{M},h}$.

So $\widetilde{\mathcal{B}}$ may be defined from $\widetilde{\mathcal{A}}^u_{[\boldsymbol{M}]}(S)$ or $\mathcal{A}_{[\widehat{\boldsymbol{M}}]}(S)$ into $\mathbb{C}[[z]]_{[\boldsymbol{M}]}$ and it is continuous when considered between the corresponding (LB), Fréchet or Banach spaces.

Surjectivity intervals

The **surjectivity intervals** defined by

$$S_{[\widehat{\boldsymbol{M}}]} := \{\gamma > 0 : \quad \widetilde{\mathcal{B}} : \mathcal{A}_{[\widehat{\boldsymbol{M}}]}(S_\gamma) \longrightarrow \mathbb{C}[[z]]_{[\boldsymbol{M}]} \text{ is surjective}\},$$
$$\widetilde{S}^u_{[\boldsymbol{M}]} := \{\gamma > 0 : \quad \widetilde{\mathcal{B}} : \widetilde{\mathcal{A}}^u_{[\boldsymbol{M}]}(S_\gamma) \longrightarrow \mathbb{C}[[z]]_{[\boldsymbol{M}]} \text{ is surjective}\}.$$

are either empty or left-open intervals having 0 as endpoint,

We have that $(\widetilde{S}^u_{[\boldsymbol{M}]})^\circ \subseteq S_{[\widehat{\boldsymbol{M}}]} \subseteq \widetilde{S}^u_{[\boldsymbol{M}]}$.

Linear and continuous operators right inverses for the asymptotic Borel map are called **extension operators**.

They can be **global**, defined from $\mathbb{C}[[z]]_{[\boldsymbol{M}]}$ into $\widetilde{\mathcal{A}}^u_{[\boldsymbol{M}]}(S)$ or $\mathcal{A}_{[\widehat{\boldsymbol{M}}]}(S)$ (with their respective (LB) or Fréchet space structures), or **local**, at the level of Banach spaces, defined from $\mathbb{C}[[z]]_{\boldsymbol{M},h}$ into some $\widetilde{\mathcal{A}}^u_{\boldsymbol{M},h'}(S)$ or $\mathcal{A}_{\widehat{\boldsymbol{M}},h'}(S)$ for suitable h' depending on h.

Additional conditions

We say $\boldsymbol{M}$ and $\boldsymbol{L}$ are **equivalent**, denoted $\boldsymbol{M} \approx \boldsymbol{L}$, if there exist $h_1, h_2 > 0$ such that for all $n \geq 0$: $h_1^{n+1} L_n \leq M_n \leq h_2^{n+1} L_n$.

If $\boldsymbol{M} \approx \boldsymbol{L}$, then the corresponding classes coincide.

$\boldsymbol{M}$ is **derivation closed (dc)** if

$$\exists C_0 > 0,\ \exists H > 1 :\ \forall n \geq 0,\quad M_{n+1} \leq C_0 H^{n+1} M_n.$$

$\boldsymbol{M}$ has **moderate growth (mg)** if

$$\exists C_0 > 0,\ \exists H > 1 :\ \forall n, k \geq 0,\ M_{n+k} \leq C_0 H^{n+k} M_n M_k.$$

Conditions (dc) and (mg) are preserved under equivalence.

If $\boldsymbol{M}$ has (mg), then $\boldsymbol{M}$ is (dc).

Gamma index

For a w.s. $\boldsymbol{M}$, Thilliez's **gamma index** is given by

$\gamma(\boldsymbol{M}) = \sup\{\gamma > 0 : (m_n/(n+1)^\gamma)_{n=0}^\infty$ is almost increasing$\} \in [0, \infty]$.

Given $\beta > 0$, we say that $\boldsymbol{m}$ satisfies the **(Schmets-Valdivia) condition** (γ_β) if there exists $A > 0$ such that for all $n \geq 0$:

$$\sum_{\ell=n}^{\infty} \frac{1}{(m_\ell)^{1/\beta}} \leq \frac{A(n+1)}{(m_n)^{1/\beta}}.$$

[J.J.-G., J. Sanz, G. Schindl (2019)] For any w.s. $\boldsymbol{M}$, we have that

$$\gamma(\boldsymbol{M}) > \beta \iff \boldsymbol{m} \text{ satisfies } (\gamma_\beta).$$

In fact, $\gamma(\boldsymbol{M})$ is the lower Matuszewska index of $\boldsymbol{m}$.

Conjecture

Let $\boldsymbol{M}$ be a weight sequence.

(a) If $\gamma(\boldsymbol{M}) = 0$, then $S_{[\widehat{\boldsymbol{M}}]} = \tilde{S}^u_{[\boldsymbol{M}]} = \emptyset$.
[H.-J. Petzsche (1988), V. Thilliez (2003), J.J.-G., J. Sanz, G. Schindl (2019)]

(b) If $\gamma(\boldsymbol{M}) = \infty$, then $S_{[\widehat{\boldsymbol{M}}]} = \tilde{S}^u_{[\boldsymbol{M}]} = (0,\infty)$ and there exist global extension operators for any opening $\gamma > 0$.
[J. Schmets, M. Valdivia (2000), J.J.-G., J. Sanz, G. Schindl (2021)]

Conjecture (Roumieu): Let $\boldsymbol{M}$ be a w.s. with $\gamma(\boldsymbol{M}) \in (0,\infty)$, then

$$S_{\{\widehat{\boldsymbol{M}}\}} = \tilde{S}^u_{\{\boldsymbol{M}\}} = (0, \gamma(\boldsymbol{M})).$$

Known results in the Roumieu case

Additional assumptions	Information
$\emptyset$	$(0, \lceil\gamma(\boldsymbol{M})\rceil - 1) \subseteq S_{\{\widehat{\boldsymbol{M}}\}} \subseteq \tilde{S}^u_{\{\boldsymbol{M}\}} \subseteq (0, \lfloor\gamma(\boldsymbol{M})\rfloor + 1]$
(dc)	$(0, \gamma(\boldsymbol{M})) \subseteq S_{\{\widehat{\boldsymbol{M}}\}} \subseteq \tilde{S}^u_{\{\boldsymbol{M}\}} \subseteq (0, \gamma(\boldsymbol{M})]$
(dc) $\wedge\ \gamma(\boldsymbol{M}) \in \mathbb{N}$	**the conjecture is true**
(mg) $\wedge\ \gamma(\boldsymbol{M}) \in \mathbb{Q}$	**the conjecture is true**
$(n!^\alpha)_{n=0}^\infty, \alpha > 0$	**the conjecture is true**

Based on: [J.P. Ramis (1978), H.-J. Petzsche (1988), J. Schmets, M. Valdivia (2000), V. Thilliez (2003), J. Sanz(2014), A. Debrouwere (2020), J.J.-G., J. Sanz, G. Schindl (2019,2021) J.J.-G., I. Miguel-Cantero, J. Sanz, G. Schindl (2023)]

Shifted moments condition

$\boldsymbol{M}$ has **shifted moments (sm)** if

$$\exists C_0 > 0,\ H > 1 :\ \forall n \geq 0,\ \log\left(\frac{m_{n+1}}{m_n}\right) \leq C_0 H^{n+1}.$$

The property (sm) is preserved under equivalence of sequences.

Let $\boldsymbol{M}$ be a sequence such that $a_0 := \inf_{n\geq 0} m_n > 0$ (e.g. if $\boldsymbol{M}$ is (lc)):

(a) (dc) implies (sm).

(b) (sm) is equivalent to $\exists C_1 > 0, H > 1 : \forall n \geq 0, \log(m_n) \leq C_1 H^n$,

Examples:
$\boldsymbol{M} = (n^{\tau n^\sigma})_n$ $(\tau > 0,\ \sigma \geq 2)$ do not satisfy (dc), but they satisfy (sm).
$\boldsymbol{M} = (q^{n^n})_n$ $(q > 1;\ M_0 := 1)$ do not satisfy (sm).

Optimal flat functions

Let $\boldsymbol{M}$ be a weight sequence. The **associated function** is defined by

$$\omega_{\boldsymbol{M}}(t) := \sup_{n\geq 0} \log\left(\frac{t^n}{M_n}\right).$$

Let $\boldsymbol{M}$ be a weight sequence, S be a sector. A function $G \in \mathcal{H}(S)$ is said to be an **$\{\boldsymbol{M}\}$-optimal flat function** if:

(1) G is $\{\boldsymbol{M}\}$−flat in S, that is:

$$\exists K_1, K_2 > 0 : \ \forall z \in S, \quad |G(z)| \leq K_1 \exp(-\omega_{\boldsymbol{M}}(K_2/|z|)).$$

(2) G is optimal, that is:

$$\exists K_3, K_4 > 0 : \ \forall x \in (0,\infty), \quad G(x) \geq K_3 \exp(-\omega_{\boldsymbol{M}}(K_4/x)).$$

Kernel functions and moments

Proposition. [J.J.-G., I. Miguel-Cantero, J. Sanz, G. Schindl (2023)] Let $\boldsymbol{M}$ be a weight sequence with $\gamma(\boldsymbol{M}) > 0$ and $0 < \gamma < \gamma(\boldsymbol{M})$. Then, there exists an optimal $\{\boldsymbol{M}\}-$flat function in S_γ.

Proposition. [J.J.-G., I. Miguel-Cantero, J. Sanz, G. Schindl (2025)] Suppose $\boldsymbol{M}$ is a weight sequence with $\gamma(\boldsymbol{M}) > 0$, G is an optimal $\{\boldsymbol{M}\}$-flat function in S. We define the kernel function $e\colon S \to \mathbb{C}$ by $e(z) = G(1/z)$ and for every $k \geq 0$ we define the *k-th moment* of the function $e(z)$ by

$$\mu(k) := \int_0^\infty t^p e(t)\, dt.$$

Then, $\boldsymbol{M}$ satisfies condition (sm) if and only if the shifted sequence $\boldsymbol{M}_{+1} := (M_{n+1})_{n=0}^\infty$ and $\boldsymbol{\mu} = (\mu(n))_{n=0}^\infty$ are equivalent.

Improved Borel-Ritt-Gevrey theorem

Theorem. [J.J.-G., I. Miguel-Cantero, J. Sanz, G. Schindl (2025)]
Let $\boldsymbol{M}$ be a weight sequence satisfying (sm) and with $\gamma(\boldsymbol{M}) > 0$, and let $\gamma > 0$ be given. Then, each of the following statements implies the next one:

(I) $\gamma < \gamma(\boldsymbol{M})$.

(II) There exists $c > 0$ such that for every $h > 0$ there exists an operator from $\mathbb{C}[[z]]_{\boldsymbol{M},h}$ into $\tilde{\mathcal{A}}^u_{\boldsymbol{M},ch}(S_\gamma)$.

(III) The Peano-Borel map $\tilde{\mathcal{B}} : \tilde{\mathcal{A}}^u_{\{\boldsymbol{M}\}}(S_\gamma) \to \mathbb{C}[[z]]_{\{\boldsymbol{M}\}}$ is surjective. In other words, $(0, \gamma] \subseteq \tilde{S}^u_{\{\boldsymbol{M}\}}$.

(IV) $(0, \gamma) \subseteq S_{\{\widehat{\boldsymbol{M}}\}}$.

In particular, one has $(0, \gamma(\boldsymbol{M})) \subseteq S_{\{\widehat{\boldsymbol{M}}\}} \subseteq \tilde{S}^u_{\{\boldsymbol{M}\}}$.

How to construct extension operators

Idea: We take G is an optimal $\{\boldsymbol{M}\}$-flat function in S_γ.
We construct the kernel function $e\colon S \to \mathbb{C}$ by $e(z) = G(1/z)$ and the corresponding moment sequence $\boldsymbol{\mu}$.

Given $h > 0$ and $\widehat{f} \in \mathbb{C}[[z]]_{\boldsymbol{M},h}$, we consider its formal Borel-like transform: We divide its coefficients by the corresponding term of the sequence $\boldsymbol{\mu}$, and thanks to (sm) we have a convergent series.

Then we apply a truncated Laplace-like transform using the kernel e to construct a function in $f \in \widetilde{\mathcal{A}}^u_{\boldsymbol{M},ch}(S_\gamma)$ such that $\mathcal{B}(f) = \widehat{f}$.

When working with the tools of Borel-Laplace $\boldsymbol{M}$-summability, this shift arises naturally, so (sm) is the appropriate condition.

New information

Additional assumptions	Information
$\emptyset$	$(0,\lceil\gamma(\boldsymbol{M})\rceil-1)\subseteq S_{\{\widehat{\boldsymbol{M}}\}}\subseteq\tilde{S}^u_{\{\boldsymbol{M}\}}\subseteq(0,\lfloor\gamma(\boldsymbol{M})\rfloor+1]$
(sm)	$(0,\gamma(\boldsymbol{M}))\subseteq S_{\{\widehat{\boldsymbol{M}}\}}\subseteq\tilde{S}^u_{\{\boldsymbol{M}\}}\subseteq(0,\lfloor\gamma(\boldsymbol{M})\rfloor+1]$
(dc)	$(0,\gamma(\boldsymbol{M}))\subseteq S_{\{\widehat{\boldsymbol{M}}\}}\subseteq\tilde{S}^u_{\{\boldsymbol{M}\}}\subseteq(0,\gamma(\boldsymbol{M})]$
(dc) $\wedge\ \gamma(\boldsymbol{M})\in\mathbb{N}$	**the conjecture is true**
(mg) $\wedge\ \gamma(\boldsymbol{M})\in\mathbb{Q}$	**the conjecture is true**
Gevrey: $(n!^{\alpha})_{n=0}^{\infty}$	**the conjecture is true**

Known results in the Beurling case

Problem: Optimal flat functions are not available.

Additional assumptions	Information
$\emptyset$	$(0,\lceil\gamma(\boldsymbol{M})\rceil-1)\subseteq S_{(\widehat{\boldsymbol{M}})}\subseteq\tilde{S}^u_{(\boldsymbol{M})}$ [J. Schmets, M. Valdivia (2000)]
(dc)	$(0,\lceil\gamma(\boldsymbol{M})\rceil-1)\subseteq S_{(\widehat{\boldsymbol{M}})}\subseteq\tilde{S}^u_{(\boldsymbol{M})}\subseteq(0,\lfloor\gamma(\boldsymbol{M})\rfloor+1]$ [J. Schmets, M. Valdivia (2000)]
(mg)	$(0,\gamma(\boldsymbol{M}))\subseteq S_{(\widehat{\boldsymbol{M}})}\subseteq\tilde{S}^u_{(\boldsymbol{M})}\subseteq(0,\gamma(\boldsymbol{M})]$ [V. Thilliez (2003), A. Debrouwere (2020,2021)]

New result for regular sequences

Theorem. [A. Debrouwere (2020)] Suppose $\widehat{\boldsymbol{M}}$ is a regular sequence ($\widehat{\boldsymbol{M}}$ is a w.s. and (dc)). The following are equivalent:

(1) The Peano-Borel map $\widetilde{\mathcal{B}}\colon \mathcal{A}_{(\widehat{\boldsymbol{M}})}(S_1) \to \mathbb{C}[[z]]_{(\boldsymbol{M})}$ is surjective.

(2) There exists a global ext. operator $U_{\boldsymbol{M}} : \mathbb{C}[[z]]_{(\boldsymbol{M})} \to \mathcal{A}_{(\widehat{\boldsymbol{M}})}(S_1)$.

(3) $\gamma(\boldsymbol{M}) > 1$.

Theorem. [J.J.-G., I. Miguel-Cantero, J. Sanz, G. Schindl (2025)] Suppose $\widehat{\boldsymbol{M}}$ is a regular sequence, and let $\gamma > 0$. Each of the following statements implies the next one:

(I) $\gamma < \gamma(\boldsymbol{M})$.

(II) There exists a global ext. operator $U_{\boldsymbol{M},\gamma} : \mathbb{C}[[z]]_{(\boldsymbol{M})} \to \widetilde{\mathcal{A}}^u_{(\boldsymbol{M})}(S_\gamma)$.

(III) The Peano-Borel map $\widetilde{\mathcal{B}} : \widetilde{\mathcal{A}}^u_{(\boldsymbol{M})}(S_\gamma) \to \mathbb{C}[[z]]_{(\boldsymbol{M})}$ is surjective.

(IV) $\gamma \leq \gamma(\boldsymbol{M})$.

New result for regular sequences

Additional assumptions	Information
$\emptyset$	$(0, \lceil\gamma(\boldsymbol{M})\rceil - 1) \subseteq S_{(\widehat{\boldsymbol{M}})} \subseteq \tilde{S}^u_{(\boldsymbol{M})}$ [J. Schmets, M. Valdivia (2000)]
(dc)	$(0, \gamma(\boldsymbol{M})) \subseteq S_{(\widehat{\boldsymbol{M}})} \subseteq \tilde{S}^u_{(\boldsymbol{M})} \subseteq (0, \gamma(\boldsymbol{M})]$
(mg)	$(0, \gamma(\boldsymbol{M})) \subseteq S_{(\widehat{\boldsymbol{M}})} \subseteq \tilde{S}^u_{(\boldsymbol{M})} \subseteq (0, \gamma(\boldsymbol{M})]$ [V. Thilliez (2003), A. Debrouwere (2020,2021)]

New result

Theorem. [J.J.-G., I. Miguel-Cantero, J. Sanz, G. Schindl (2025)]
Let $\boldsymbol{M}$ be a weight sequence with $\gamma(\boldsymbol{M}) > 0$ and that satisfies (sm), and $0 < \gamma < \gamma(\boldsymbol{M})$ be given. Then, the Peano-Borel map

$$\mathcal{B} : \tilde{A}^u_{(\boldsymbol{M})}(S_\gamma) \to \mathbb{C}[[z]]_{(\boldsymbol{M})}$$

is surjective, and so $(0, \gamma(\boldsymbol{M})) \subseteq \tilde{S}^u_{(\boldsymbol{M})}$.

Idea: Given $\widehat{f} \in \mathbb{C}[[z]]_{(\boldsymbol{M})}$, we construct, using Thilliez's technique and Chaumat-Chollet sequences, a weight sequence $\boldsymbol{N}$ that satisfies (sm), $\gamma(\boldsymbol{N}) > \gamma$, $\tilde{A}^u_{\{\boldsymbol{N}\}}(S_\gamma) \subseteq \tilde{A}^u_{(\boldsymbol{M})}(S_\gamma)$ and such that $\widehat{f} \in \mathbb{C}[[z]]_{\{\boldsymbol{N}\}}$. We conclude by applying the theorem for the Roumieu case.

New results for the Beurling case

Additional assumptions	Information
$\emptyset$	$(0, \lceil \gamma(\boldsymbol{M}) \rceil - 1) \subseteq S_{(\widehat{\boldsymbol{M}})} \subseteq \tilde{S}^u_{(\boldsymbol{M})}$ [J. Schmets, M. Valdivia (2000)]
(sm)	$(0, \gamma(\boldsymbol{M})) \subseteq S_{(\widehat{\boldsymbol{M}})} \subseteq \tilde{S}^u_{(\boldsymbol{M})}$
(dc)	$(0, \gamma(\boldsymbol{M})) \subseteq S_{(\widehat{\boldsymbol{M}})} \subseteq \tilde{S}^u_{(\boldsymbol{M})} \subseteq (0, \gamma(\boldsymbol{M})]$
(mg)	$(0, \gamma(\boldsymbol{M})) \subseteq S_{(\widehat{\boldsymbol{M}})} \subseteq \tilde{S}^u_{(\boldsymbol{M})} \subseteq (0, \gamma(\boldsymbol{M})]$ [V. Thilliez (2003), A. Debrouwere (2020,2021)]

THANK YOU VERY MUCH
FOR YOUR ATTENTION!

THANK YOU VERY MUCH
FOR YOUR ATTENTION!

¡FELIZ CUMPLEAÑOS!