A Jubilee Theorem for Pepe Bonet

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The contributors

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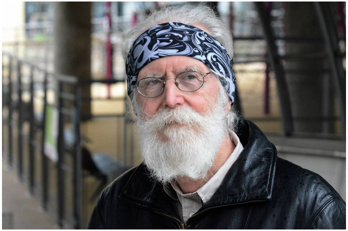
Michel Talagrand (Abel prize 2024) meeting José Bonet in 1986



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- Stochastic Processes- Mathematical Physics

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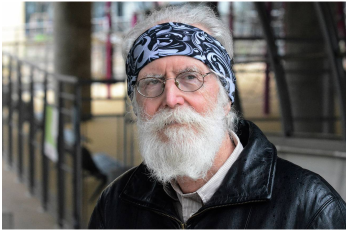

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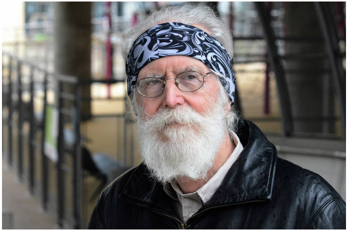

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Theorem (J.O)

For a web-compact topological space X; in particular por every K-analytic or weakly countably determined space, the function space $C_p(X)$ is angelic

- Overwolfach, plagiarism, THANKS FOR LIFE PEPE!!!!

- This result was later devolopped by different authors in several books,.... J. Kakol, M. López Pellicer, W. Kubis, ...

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LUR norms

Definition

If $(E, \|\cdot\|)$ is a normed space, the norm $\|\cdot\|$ is said to be **locally uniformly rotund** (**LUR**, for short) if

$$\left[\lim_n \left(2\|x\|^2 + 2\|x_n\|^2 - \|x + x_n\|^2\right) = 0\right] \Rightarrow \lim_n \|x - x_n\| = 0 \tag{1}$$

for any sequence $\{x_n\}_{n=1}^{\infty}$ and any x in E. On the other hand, $\|\cdot\|$ is said to be **rotund** (**R**, for short), or **strictly convex**, if

$$\left[2\|x\|^2 + 2\|y\|^2 - \|x + y\|^2 = 0\right] \Rightarrow \|x - y\| = 0 \tag{2}$$

An equivalent, more geometrical, definition of the LUR property of the norm reads: If $\{x, x_1, x_2, \dots\} \subset S_E$ and $\|x + x_n\| \to 2$, then $\|x - x_n\| \to 0$.

$$Q_{\|\cdot\|}(x, y) := 2\|x\|^2 + 2\|y\|^2 - \|x + y\|^2. \tag{3}$$

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Definition

A family of subsets $\mathcal{N}$ in a topological space (T, τ) is a **network** for the topology τ if for every $W \in \tau$ and every $x \in W$, there is some $N \in \mathcal{N}$ such that $x \in N \subset W$.

A central result for the theory is the following one due to Moltó, Troyanski, Raja and myself:

Theorem (Slicely Network)

Let E be a normed space, F a norming subspace of E^* and $\mathcal{H}$ the family of all $\sigma(E, F)$-open half-spaces in E. Then E admits a $\sigma(E, F)$-lower semicontinuous equivalent **LUR** *norm if, and only if, there is a sequence $\{A_n\}_{n=1}^{\infty}$ of subsets of E such that for every $\epsilon > 0$ and every $x \in E$ there is a $\sigma(E, F)$-open half space H and an integer n such that*

$$x \in A_n \cap H \text{ and } \|\cdot\|\text{-diam}(A_n \cap H) < \epsilon.$$

Theorem (Open Localization Theorem)

Let A be a bounded subset in E and $\mathcal{C} = \{\Theta_i : i \in I\}$ be $\sigma(E, F)$-closed convex subsets of E.
Then, there is an equivalent $\sigma(E, F)$-lower semicontinuous norm $\|\cdot\|_{\mathcal{C},A}$ such that:
If $x \in A \setminus \Theta_{i_0}$ for some $i_0 \in I$, and $\{x_n\}_{n=1}^{\infty}$ is a sequence in E such that $\lim_n Q_{\|\cdot\|_{\mathcal{C},A}}(x_n, x) = 0$, then there is a sequence $\{i_n\}_{n=1}^{\infty}$ in I such that:
There is $n_0 \in \mathbb{N}$ such that $x \in A \setminus \Theta_{i_n}$ for each $n \geq n_0$.
Moreover, if for some $n \geq n_0$ we have $x_n \in A$, then $x_n \in A \setminus \Theta_{i_n}$.

Theorem (Open localization plus aproximation theorem)

Let A be a bounded subset in E and $\mathcal{C} := \{\Theta_i : i \in I\}$ be a family of convex and $\sigma(E,F)$-closed subsets of E.

Then there is an equivalent $\sigma(E,F)$-lower semicontinuous norm $\|\cdot\|_{\mathcal{C},A}$ on E such that given $x \in A \setminus \Theta$ for some $\Theta \in \mathcal{C}$ and a sequence $\{x_n\}_{n=1}^{\infty}$ in E with $\lim_n Q_{\|\cdot\|_{\mathcal{C},A}}(x_n, x) = 0$, then there is a sequence $\{i_n\}_{n=1}^{\infty}$ in I verifying the two following properties:

- (i) *There is $n_0 \in \mathbb{N}$ with $x \in A \setminus \Theta_{i_n}$ for each $n \geq n_0$. Moreover, if $x_n \in A$ for some $n \geq n_0$, then $x_n \in A \setminus \Theta_{i_n}$.*
- (ii) *Additionally we will still have the following approximation: For every $\delta > 0$ there is some $n_\delta \in \mathbb{N}$ such that*

$$x, x_n \in \overline{\operatorname{co}\left(A \setminus \Theta_{i_n}\right) + \delta B_E}^{\sigma(E,F)} \text{ for all } n \geq n_\delta. \tag{4}$$

Theorem (Δ-Convex Networking)

The following are equivalent:
(i) *E admits a $\sigma(E,F)$-lower semicontinuous equivalent LUR norm.*
(ii) *If $\{A_n\}_{n=1}^{\infty}$ denotes the sequence of balls centered at 0 and having rational radius, and $\mathcal{H}$ denotes the family of all open half-spaces defined by elements in F, then the family of sets $\{A_n \cap H : \ H \in \mathcal{H}, \ n \in\}$ is a network for the norm topology in E.*
(iii) *There is a sequence $\{A_n\}_{n=1}^{\infty}$ of $\sigma(E,F)$-closed convex subsets of E such that the family of sets*

$$\{A_n \setminus \Theta : \ \Theta \in \mathcal{C}, n \in \mathbb{N}\}$$

is a network for the norm topology in E.
(iv) *There is a sequence $\{A_n\}_{n=1}^{\infty}$ of subsets of E such that the family of sets $\{A_n \setminus \Theta : \ \Theta \in \mathcal{C}, n \in \mathbb{N}\}$ is a network for the norm topology in E.*

Theorem (A new main LUR result)

A Banach space E, with a norming subspace $F \subset E^*$, has an equivalent $\sigma(E, F)$-lower semicontinuous LUR norm if, and only if:
There is a sequence $\{A_n\}_{n=1}^{\infty}$ of subsets of E such that, given any $x \in E$ and $\epsilon > 0$, there is a $\sigma(E, F)$-open half-space H and $n \in \mathbb{N}$ with

$$x \in H \cap A_n \subset S_n^H + B(0, \epsilon)$$

where S_n^H is a separable subset of E .

Corollary

A Banach space E with a norming subspace $F \subset E^*$, has an equivalent $\sigma(E, F)$-lower semicontinuous LUR norm if, and only if, it has anoher one with separable denting faces of its closed unit ball.

New Examples

- This result completely solves four problems asked by Moltó, Troyanski, Valdivia and myself. It is an extension of Troyanski's fundamental results (see Chapter IV in Deville -Godefroy -Zizler book), as well as Raja's theorems in LUR renormings and García-Oncina-Troyanski and J.O.
- Banach spaces $C(K)$, *where* K *is a Rosenthal compact space* $K \subset \mathbb{R}^\Gamma$ *(i.e., a compact space of Baire one functions on a Polish space* Γ*,) with at most countably many discontinuity points for every* $s \in K$, (question asked by R. Haydon, A.Moltó and myself)

Question: Characterize those Banach spaces which have an equivalent strictly convex norm.

It is easily verified that every separable Banach space has an strictly convex norm. The same is true for a general WCG space. On the other hand, it was shown by Day that there exist Banach spaces which do not have an equivalent strictly convex norm.

Some conjectures concerning a possible answer to the question were shown to be false by Dashiell and Lindenstrauss.

This results shows that even for $C(K)$ spaces it seems to be a delicate and presumably difficult question to decide under which condition there exists an equivalent strictly convex norm.

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Definition

We say that a topological space (X, τ) **is a $T_0(*)$-space** or that the topology τ **is** $T_0(*)$ if there is a system $\{\mathcal{W}_n : \ n \in \mathbb{N}\}$, where each $\mathcal{W}_n$ is a family of open sets, such that for $x \neq y$ there is some $p \in \mathbb{N}$ for which either we have

$$y \notin \mathrm{Star}(x, \mathcal{W}_p) \neq \emptyset \text{ or } x \notin \mathrm{Star}(y, \mathcal{W}_p) \neq \emptyset.$$

For a family $\mathcal{F}$ of subsets of X, let us remind you:

$$\mathrm{Star}(x, \mathcal{F}) := \bigcup\{F : x \in F \in \mathcal{F}\}.$$

Systems $\{\mathcal{W}_n : n \in \mathbb{N}\}$ are said to $T_0(*)$**- separate points of E.** For a system $\{\mathcal{G}_n : n \in \mathbb{N}\}$, where each $\mathcal{G}_n$ consists of functions from E into $\mathbb{R}$, we say that $\{\mathcal{G}_n : n \in \mathbb{N}\}$ $T_0(*)$**- separates points of E** whenever the system $\{\mathcal{O}_n : n \, in \mathbb{N}\}$ $T_0(*)$-separates points of E, where $\mathcal{O}_n := \{O_g : \ g \in \mathcal{G}_n\}$ for $n \in \mathbb{N}$, and

$$O_g := \{x \in E : \ g(x) > 0\}. \tag{5}$$

A Solution

Theorem (Strictly Convex Renorming)

Let E be a normed space with a norming subspace $F \subset E^*$. Then E admits an equivalent $\sigma(E, F)$-lower semicontinuous and strictly convex norm if, and only if, there are families $\mathcal{G}_n$, $n \in \mathbb{N}$, of $\sigma(E, F)$-lower semicontinuous quasi-convex functions defined on E such that the system $\{\mathcal{G}_n : n \in \mathbb{N}\}$ $T_0(*)$-separates points of E.

R. Smith, S. Troyanski and J.O. proved this result where the functions g above are in F and the open sets O_g are open half spaces.

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The dual case. A completely new result

Theorem (Strictly Convex Renorming)

Let E be a normed space. Then E^* admits an equivalent $\sigma(E^*, E)$-lower semicontinuous and strictly convex norm if, and only if, $(E, \sigma(E^*, E))$-topology is a $T_0(*)$-space. In particular weak- homeomorphismes preserve dual strictly convex renormings.*

The former result answers a recent question by R. Smith in J. Math. Analysis Applications, where a proof for Asplund spaces is given.

Previous approaches with S. Ferrari give us proofs in case the dual unit sphere S_{E^*} provide us a w$^* - G_\delta$ diagonal in $S_{E^*} \times S_{E^*}$.

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General answer to Lindenstrauss' question

Theorem

Let K be a compact space where every separable subset is metrizable (i.e. a monolithic compact space). Then the Banach space $(S(K), \|\cdot\|_\infty)$ of all continuous functions on K with separable support admits a pointwise lower semicontinuous and locally uniformly rotund renorming.Moreover the Banach space $(C(K), \|\cdot\|_\infty)$ has an equivalent strictly convex norm.

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Historical context of our research

LUR-renorm. $\rightarrow$ Kadec-renorm.$\rightarrow$ Descriptive space$\rightarrow$ weakly Cech-analytic $\rightarrow$ σ-fragmentable

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M. Jimenez, J.P. Moreno, L. Oncina, F. Garcia, S. Lajara, A. Pallarés, A. Avilés, S. Ferrari, L.C. García Girola, B. Cascales, M. Raja, A. Moltó, M. López Pellicer, M. Valdivia, A.J. Guirao, V. Montesinos, P.Hájek, T. Russo, S. Dantas, G. Martínez Cervantes, A. Rueda $\cdots$, etc.... Bulgarian-Spanish School in Renorming Theory originated inside Valencia Seminar

Historical context of our research

LUR-renorm. $\rightarrow$ Kadec-renorm.$\rightarrow$ Descriptive space$\rightarrow$ weakly Cech-analytic $\rightarrow$ σ-fragmentable

P. Enflo, G. Pisier and M. Talagrand
J.E. Jayne, I. Namioka and C.A. Rogers
Z. Frolík, P. Habala, R. Hansell, P. Holicky, P. Hájek, W. Kakol
R. Haydon and R. Smith
S. Troyanski and P. Kenderov
R. Deville, G. Godefroy and V. Zizler
M. Fabian, W. Kubis, O. Kalenda, S. Argyros, S. Mercourakis, S. Negrepontis and J. Vanderwerf
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MAIN QUESTIONS:

- Does every Banach space E with the RNP admits an equivalent **R** norm?
- Does every Banach space with Fréchet differentiable norm admits an equivalent **R** norm?

Thanks a lot for your attention

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Thanks a lot for your attention

A sunny Haydon forest

Theorem

For every monolithic compact space K the space $C(K)$ admits an equivalent strictly convex norm

Theorem (Jubilee present for Pepe)

*Every Banach space with a Fréchet differentiable norm admits an equivalent **LUR** norm*

A sunny Haydon forest

Theorem

For every monolithic compact space K the space $C(K)$ admits an equivalent strictly convex norm

Theorem (Jubilee present for Pepe)

Every Banach space with a Fréchet differentiable norm admits an equivalent **LUR** *norm*